  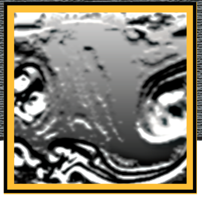

Stabilized Conservative Level Set Method with Adaptive Wavelet-based Mesh Refinement

Navid Shervani-Tabar,   Oleg V. Vasilyev

Department of Mechanical Engineering
University of Colorado at Boulder

DEPARTMENT OF MECHANICAL ENGINEERING
MULTI-SCALE MODELING & SIMULATION LABORATORY

- **Level Set Method: Sethian's formulation***

$$\phi(\mathbf{x},t) = I(\mathbf{x},t) \min_{\mathbf{y}\in\Gamma(t)} \|\mathbf{x}-\mathbf{y}\|_2$$

$$I(\mathbf{x},t) = \begin{cases} 1, & \text{if } \mathbf{x} \in \Omega(\mathrm{t}) \\ -1, & \text{if } \mathbf{x} \notin \Omega(\mathrm{t}) \end{cases}$$

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$$\frac{\partial\tilde{\phi}}{\partial\tau} + \mathrm{sgn}(\phi)(|\nabla\tilde{\phi}| - 1) = 0 \qquad \tilde{\phi}(\mathbf{x},0) = \phi(\mathbf{x},t)$$

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- **Problems and Challenges**
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  - *Accuracy*

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  - *Mass Conservation*
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- **Goals**
  - *Overcome LS volume conservation problem*
  - *Improve accuracy*

- **Conservative Level Set: Olsson's formulation***

$$\psi(\mathbf{x},t) = \frac{1}{2}\left(\tanh\left(\frac{\phi(\mathbf{x},\mathrm{t})}{2\epsilon}\right) + 1\right)$$

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$$\frac{\partial\psi}{\partial t} + \nabla\cdot(\psi\mathbf{u}) = 0$$

$$\frac{\partial\psi}{\partial\tau} + \nabla\mathbf{f}(\psi) = 0$$

$$\mathbf{f} = \psi(1-\psi)\mathbf{n} \qquad \mathbf{n} = \frac{\nabla\psi}{|\nabla\psi|}$$

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$$\frac{\partial\psi}{\partial\tau}=-\nabla\mathbf{f}(\psi)+\epsilon\Delta\psi$$

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$$\frac{\partial\psi}{\partial\tau} + \nabla\mathbf{f}(\psi) = 0 \qquad \mathbf{f} = \psi(1-\psi)\mathbf{n} \qquad \mathbf{n} = \frac{\nabla\psi}{|\nabla\psi|}$$

$$\frac{\partial\psi}{\partial\tau} = -\nabla\mathbf{f}(\psi) + \epsilon\Delta\psi$$

$$\frac{\partial\psi}{\partial\tau} + \nabla\cdot(\psi(1-\psi)\mathbf{n}) = \nabla\cdot(\epsilon\nabla\psi)$$

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- **Diffusion direction***

$$\frac{\partial \psi}{\partial \tau} = -\nabla \cdot (\psi(1-\psi)\mathbf{n}) + \nabla \cdot (\epsilon(\nabla \psi \cdot \mathbf{n})\mathbf{n})$$

$$\mathbf{n} = \frac{\nabla \psi}{|\nabla \psi|}$$

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- **Normal vector**

$$\phi(\mathbf{x}) = \epsilon \ln\left(\frac{\psi(\mathbf{x})}{1-\psi(\mathbf{x})}\right)$$

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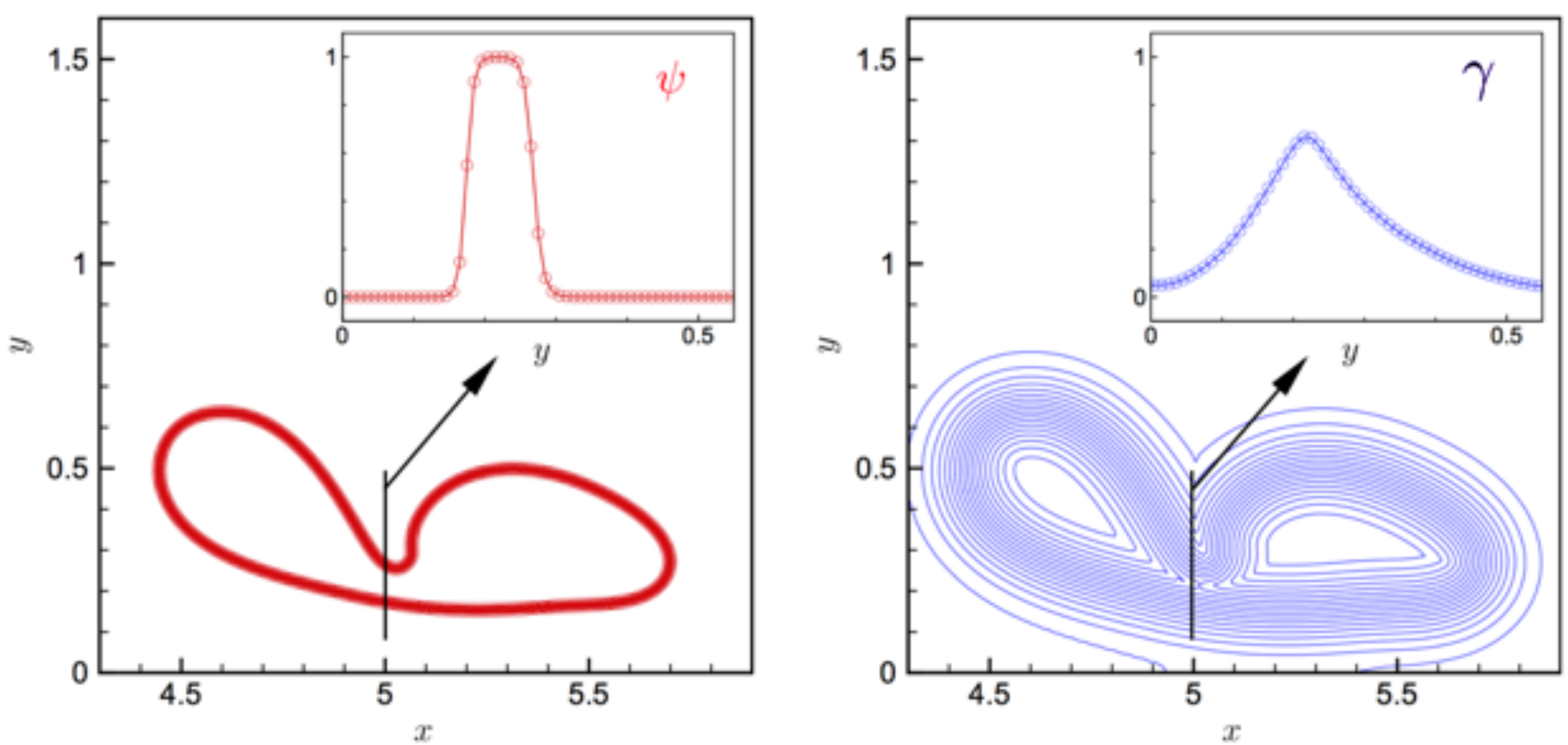
- **Using Fast Marching Method***

* *E. Olsson, G. Kreiss, S. Zahedi, A conservative level set method for two phase flow ii, J. Comput. Phys. 225 (2007) 785–807.*

* *O. Desjardins, V. Moureau, H. Pitsch, An accurate conservative level set/ghost fluid method for simulating turbulent atomization, J. Comput. Phys. 227 (2008) 8395–8416.*

- **Mapping ψ field on γ ***

$$\gamma = \frac{\psi^\alpha}{\psi^\alpha + (1-\psi)^\alpha}, \qquad \text{for} \quad \alpha < 1$$



* R. K. Shukla, C. Pantano, J. B. Freund, An interface capturing method for the simulation of multi-phase compressible flows, J. Comput. Phys. 229 (2010) 74117439.

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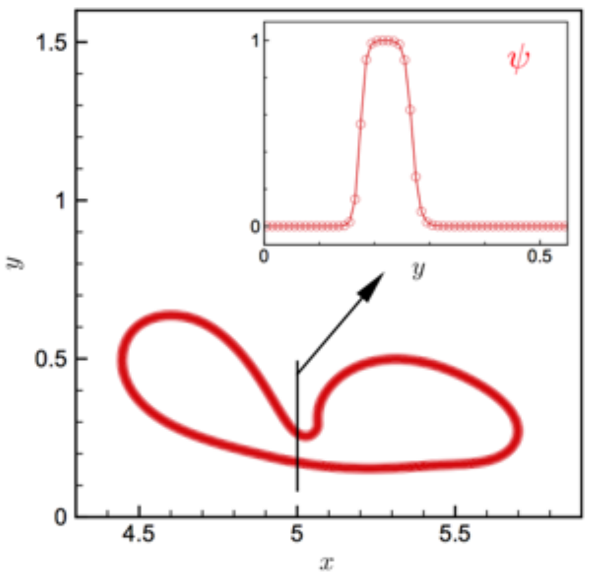

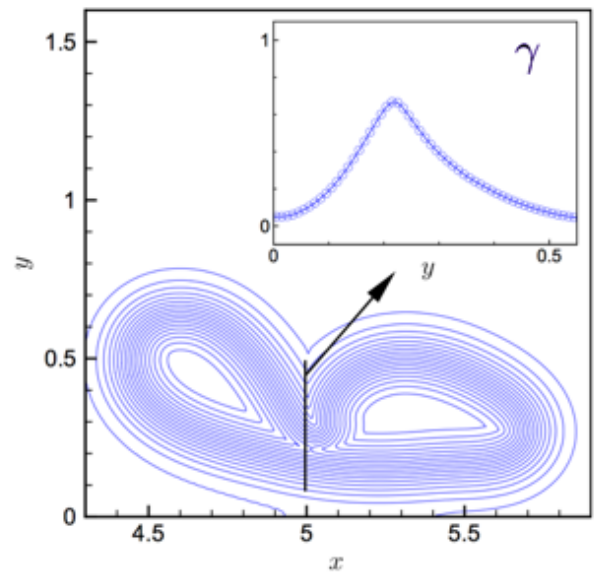

- **Coupled LS and CLS***

$$\phi(\mathbf{x}) = \epsilon \ln\left(\frac{\psi(\mathbf{x})}{1-\psi(\mathbf{x})}\right)$$

* R. K. Shukla, C. Pantano, J. B. Freund, An interface capturing method for the simulation of multi-phase compressible flows, J. Comput. Phys. 229 (2010) 74117439.
* L. Zhao, X. Bai, T. Li, J. J. R. Williams, Improved conservative level set method, Int. J. Numer. Meth. Fluids 75 (2014) 575–590.

Improvements on Conservative Level Set

- **Challenges**
  - *Ill-defined normal vector*
  - *High computational expense*
  - *Potential discontinuity*

- **Stabilized Conservative Level Set:**

$$\psi(\mathbf{x},t) = \frac{1}{2}\left(\tanh\left(\frac{\phi(\mathbf{x},\mathrm{t})}{2\epsilon}\right) + 1\right)$$

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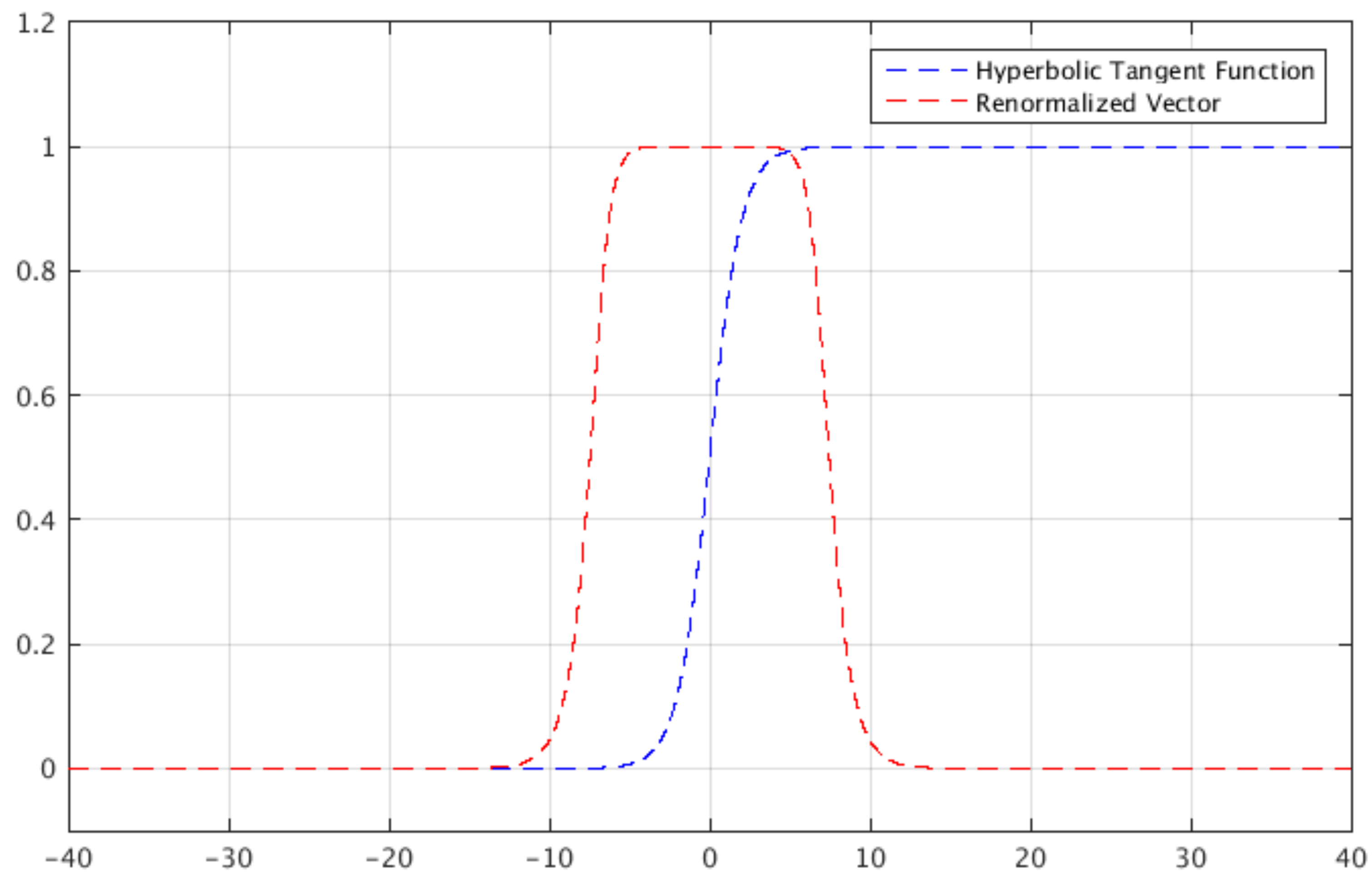

Department of Mechanical Engineering
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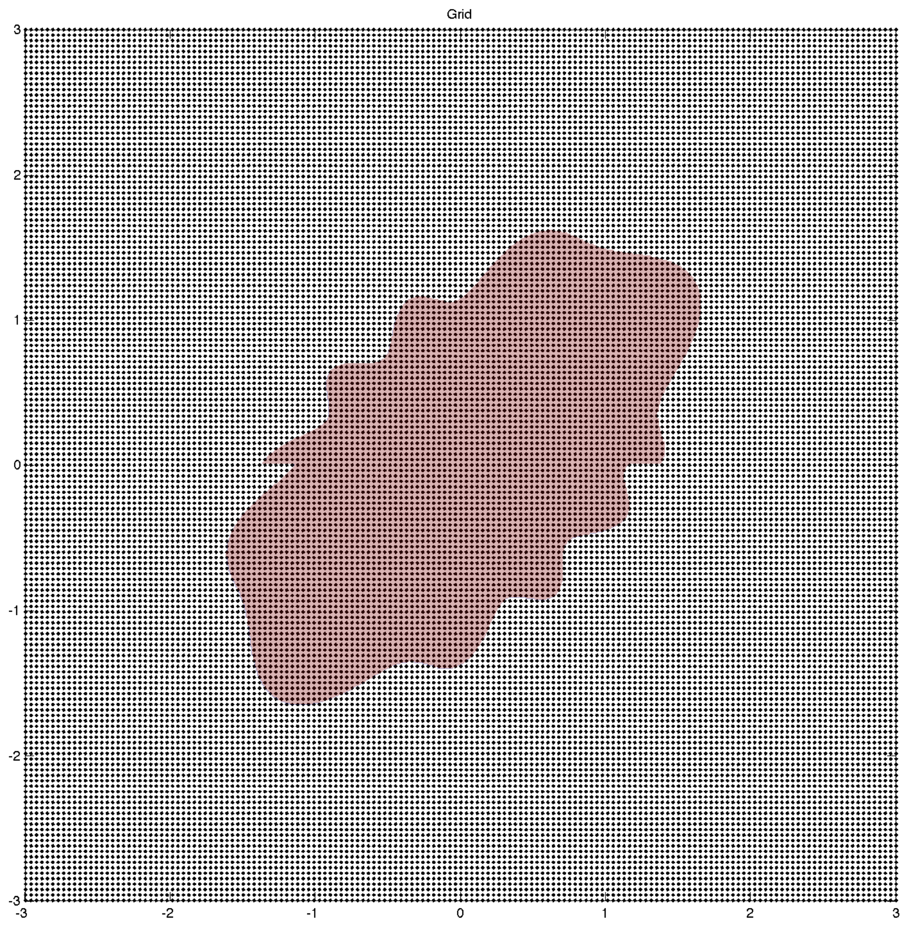
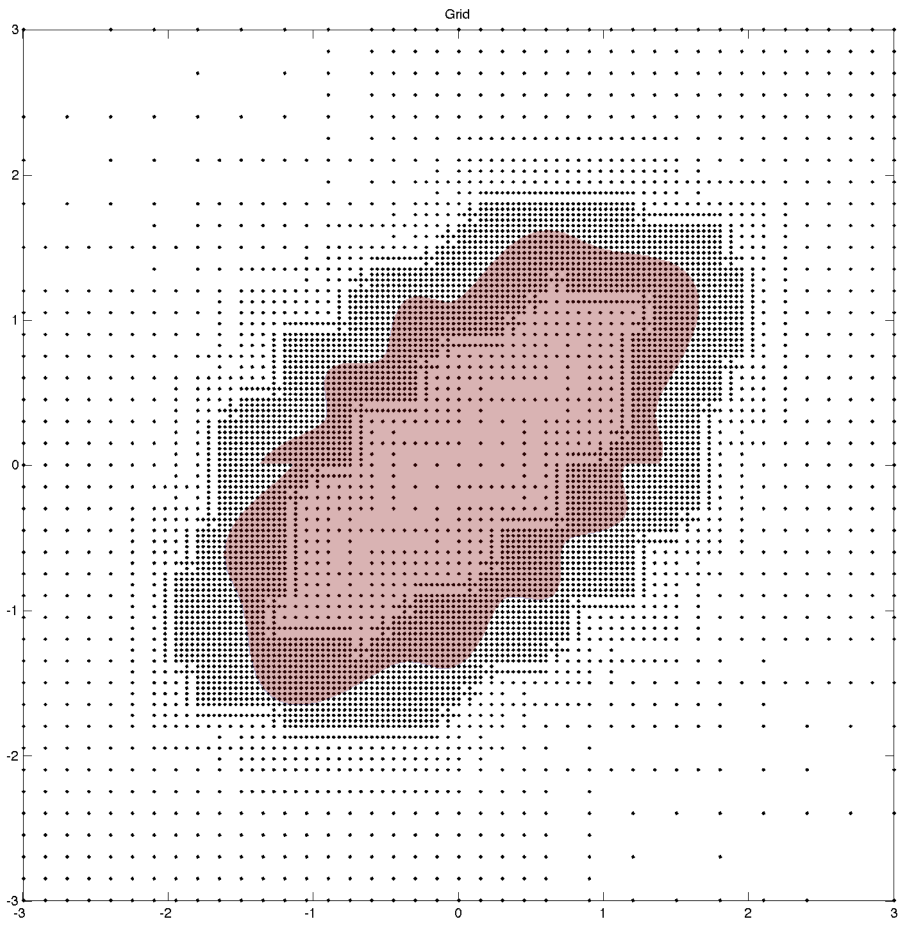
- **Grid Adaptation:**

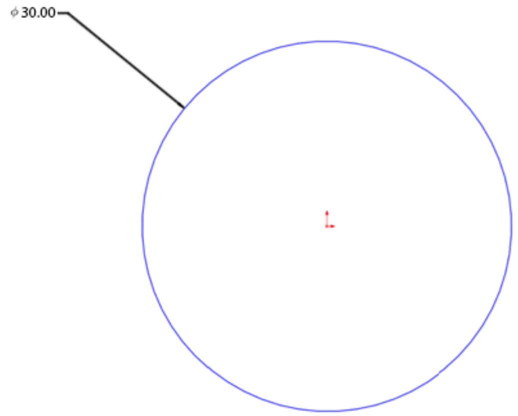




• Rotational Disk

$$\frac{\partial \psi}{\partial t} + \nabla \cdot (\psi \mathbf{u}) = 0$$

$$\begin{aligned} \mathbf{u}_x &= -2\pi y, \\ \mathbf{u}_y &= 2\pi x. \end{aligned}$$



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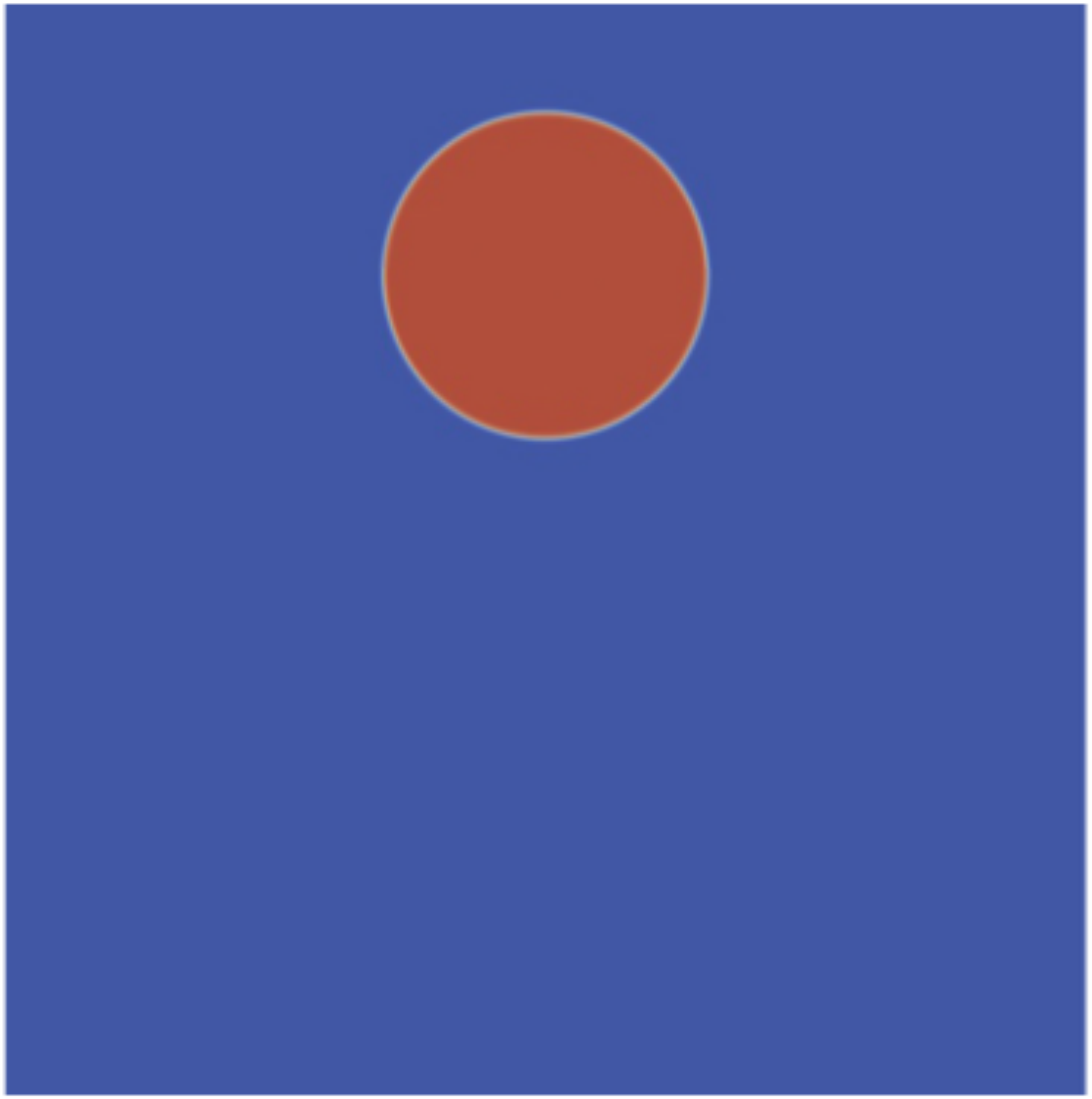

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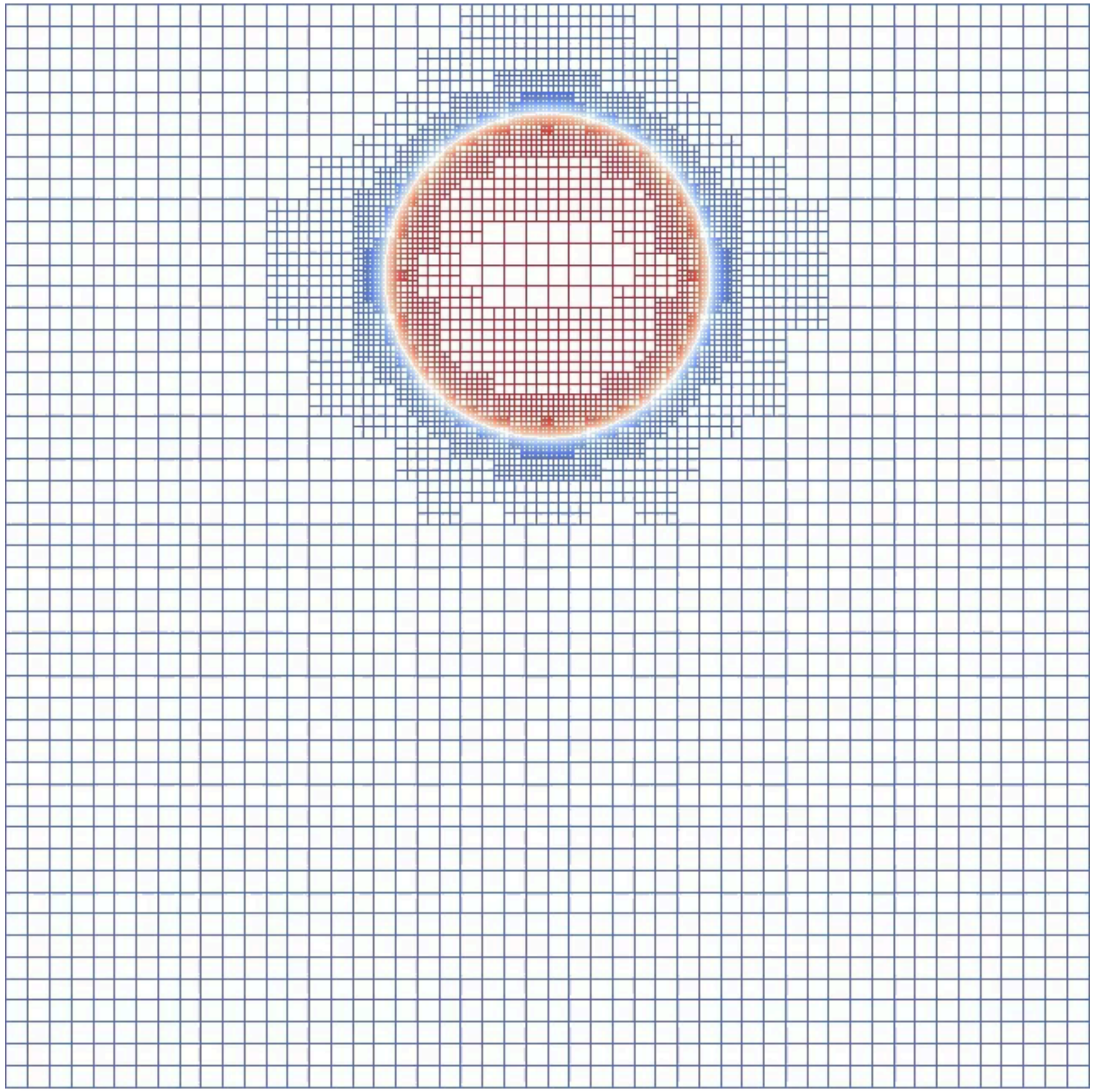

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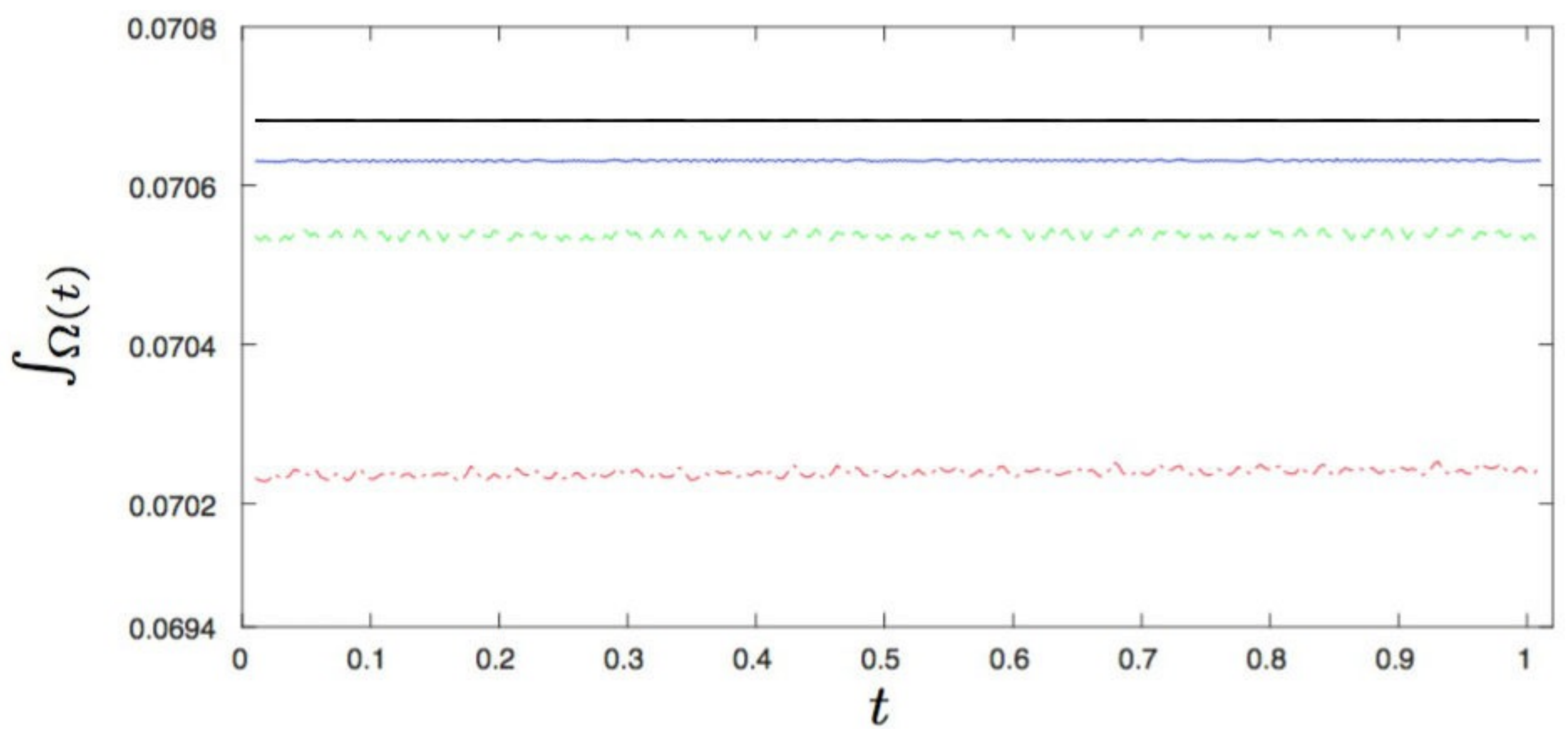
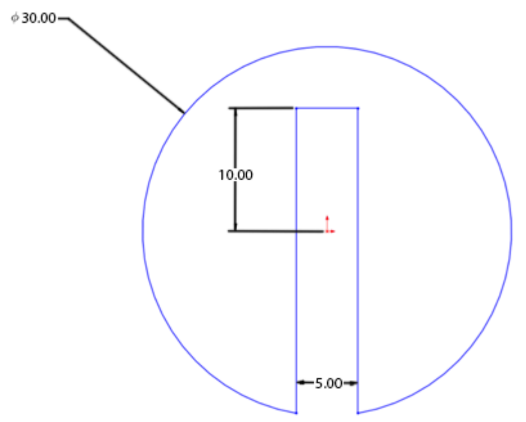


ϵ	Δx	Maximum deviation %
3.6×10^{-2}	2.4×10^{-2}	0.0202
1.8×10^{-2}	1.2×10^{-2}	0.0123
0.9×10^{-2}	0.6×10^{-2}	0.0033

- **Zalesak's Disk**



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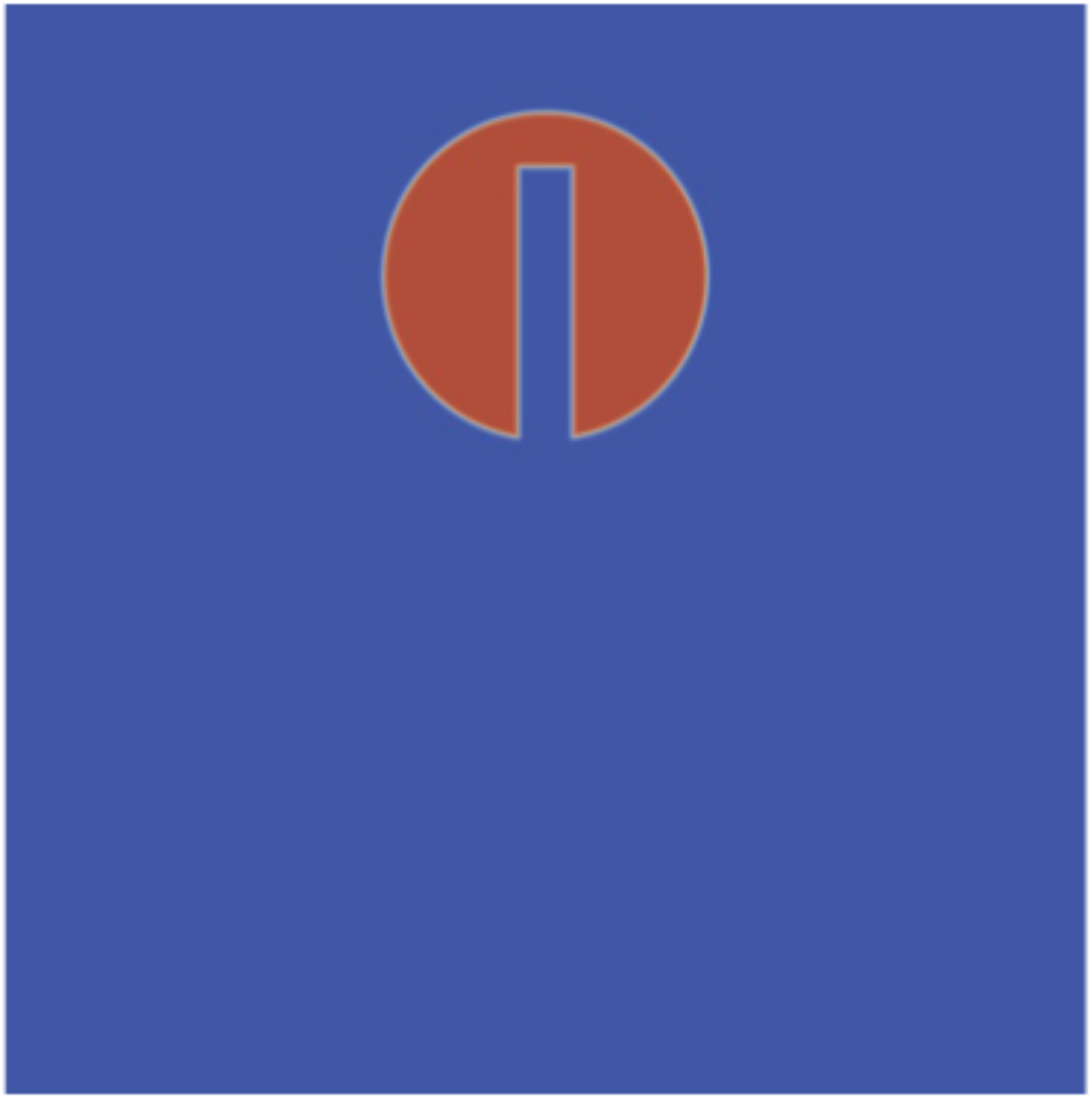

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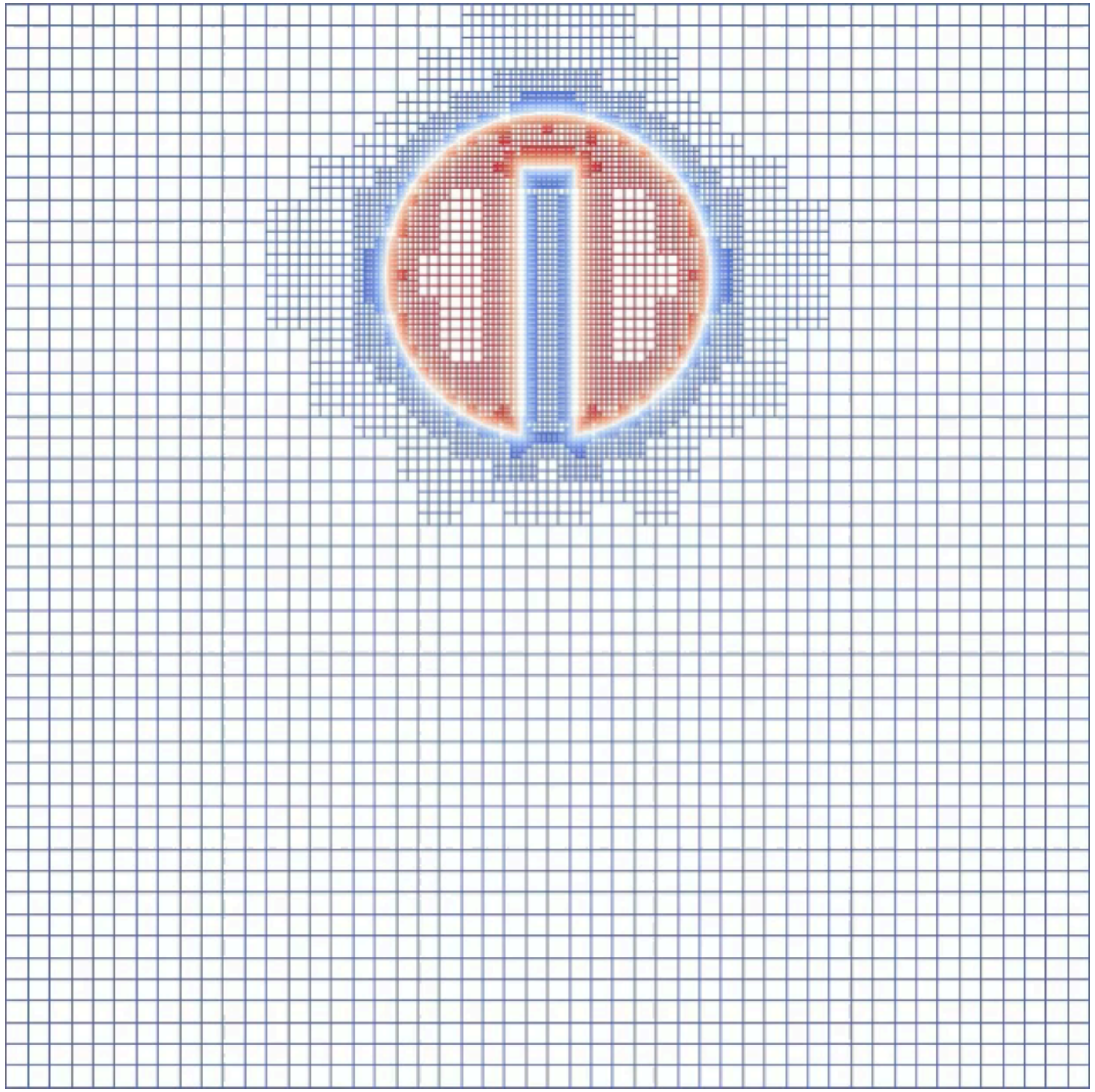
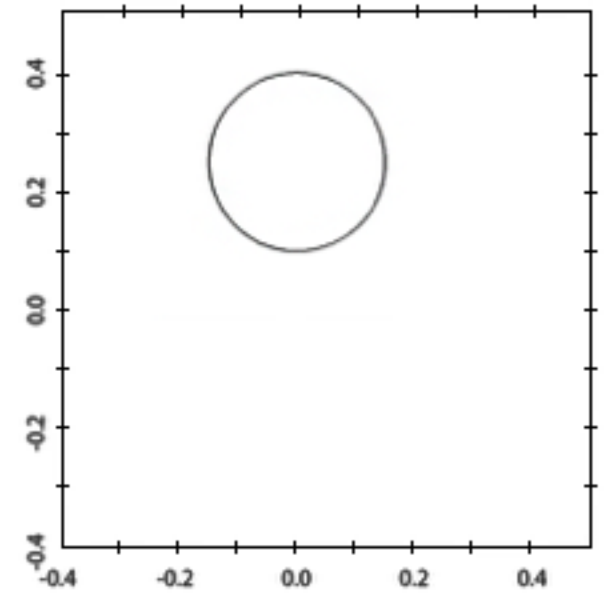
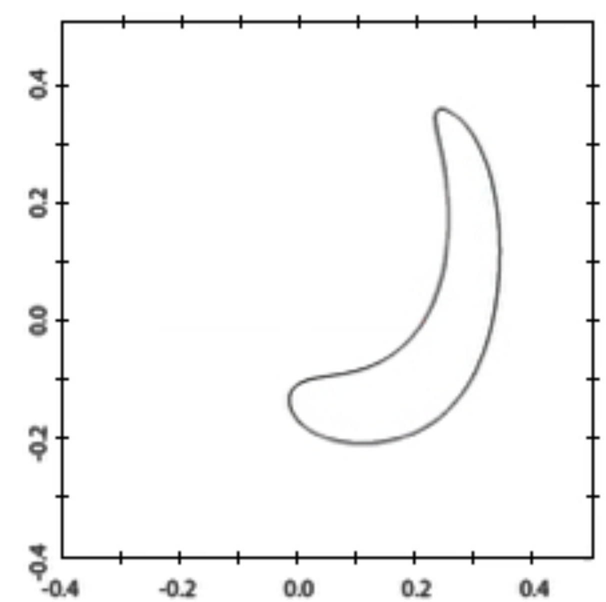


- **Vortex drop**

$$\frac{\partial \psi}{\partial t} + \nabla \cdot (\psi \mathbf{u}) = 0$$

$$\begin{aligned} \mathbf{u}_x &= \quad 2\sin^2(\pi(x - x_{\min}))\sin(\pi y)\cos(\pi y)\cos(\frac{\pi t}{T}), \\ \mathbf{u}_y &= -2\sin^2(\pi(y - y_{\min}))\sin(\pi x)\cos(\pi x)\cos(\frac{\pi t}{T}). \end{aligned}$$





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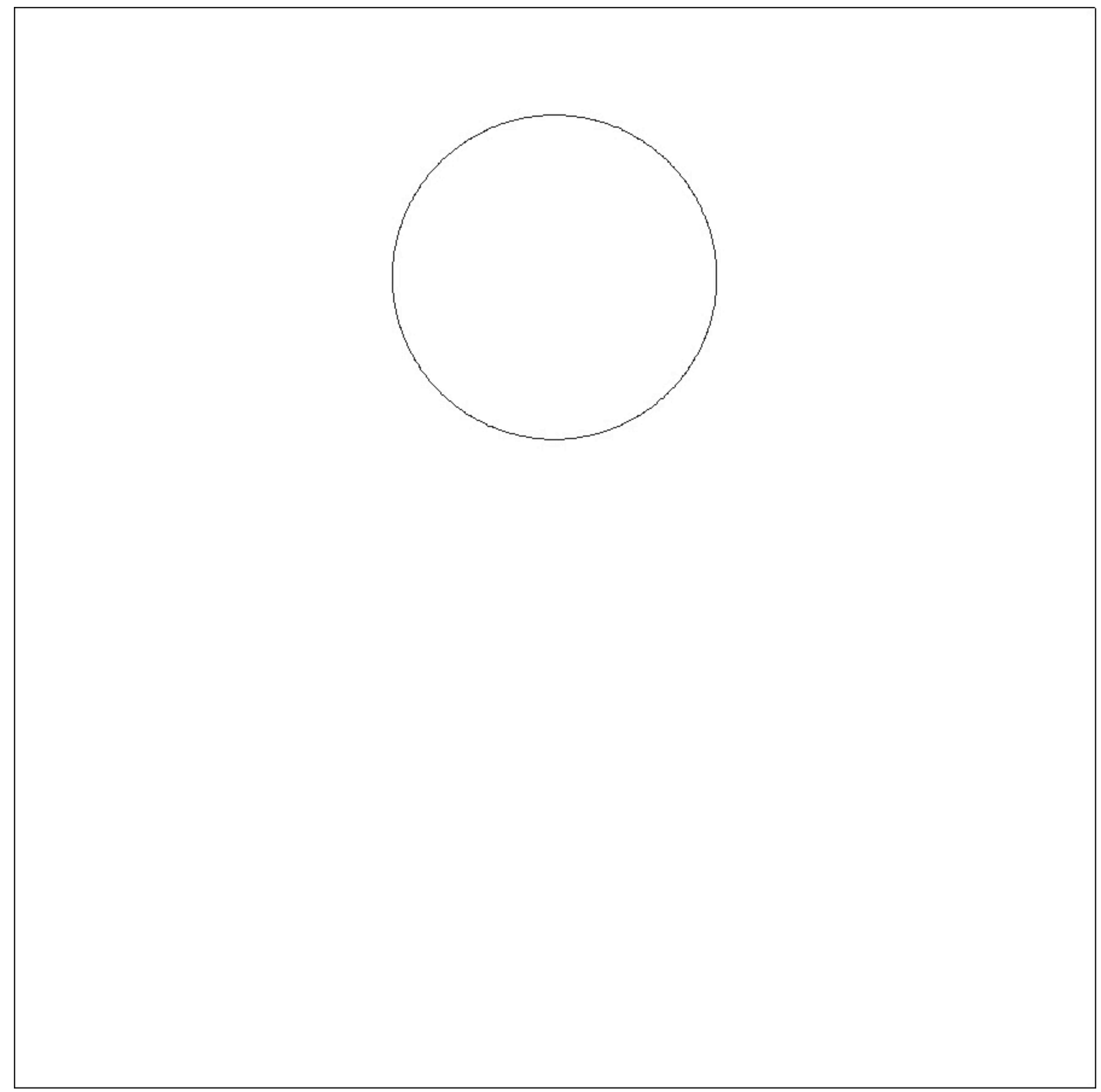
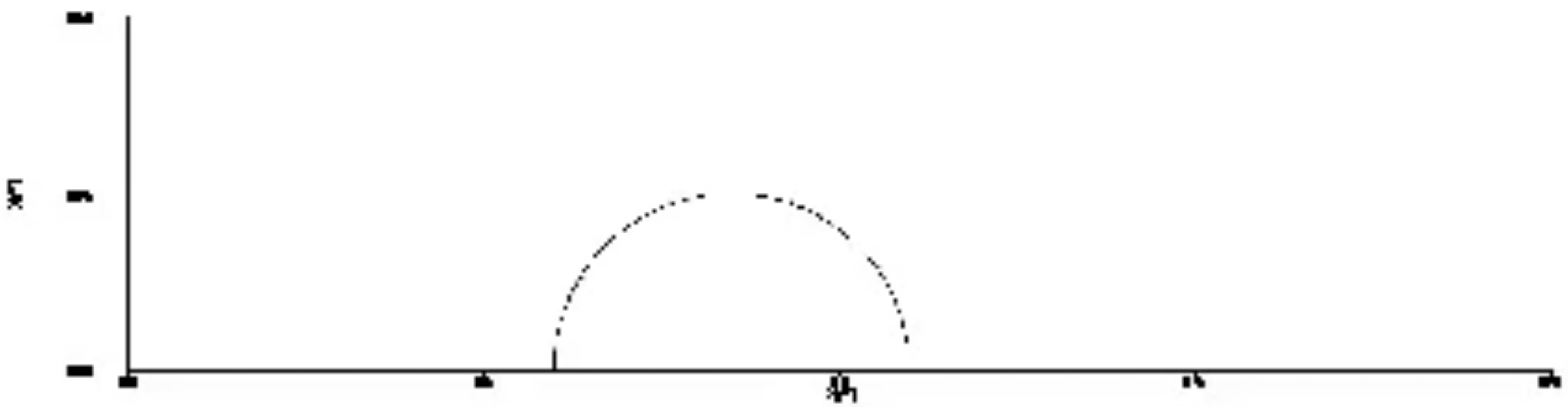


- **Based on non-unit normal vector.**
- **General, robust, and easy to implement.**
- **SCLS in combination with adaptive wavelet-based mesh refinement**
- **Accurately advect corners, sharp angles, and resolve thin filaments, while preserving excellent conservation properties of the conservative level set method.**

- **Applications to Multiphase flow:**
    - Shock-Bubble Interaction:



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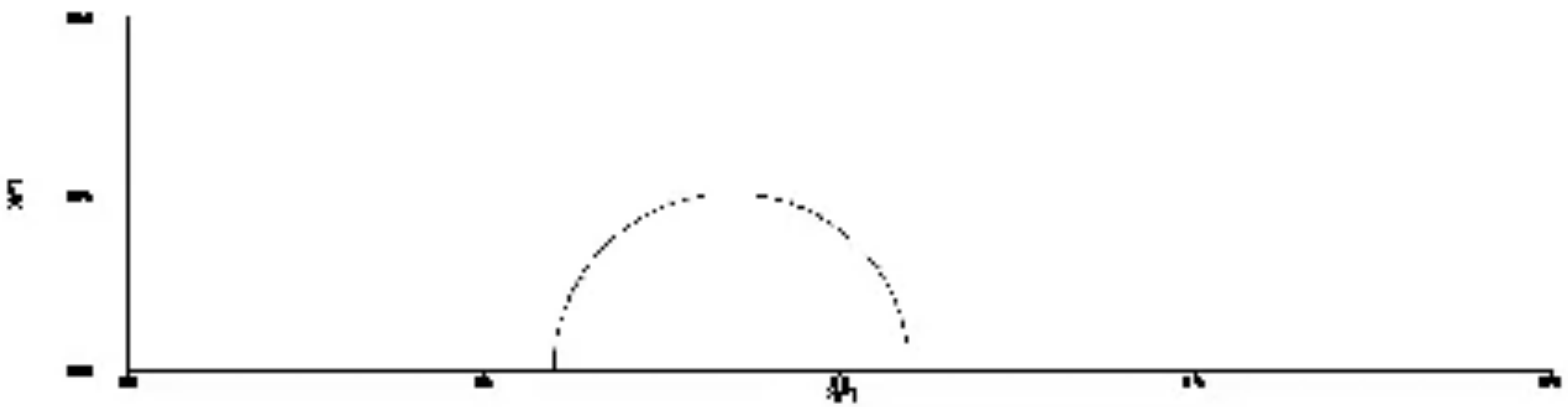

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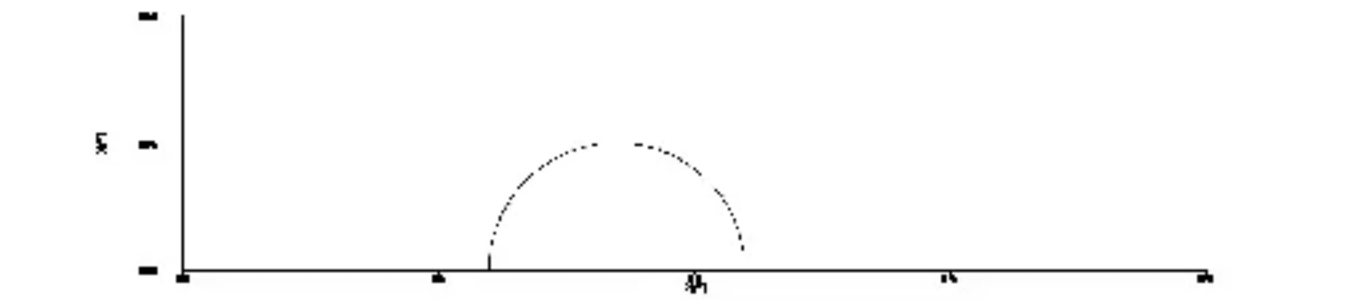

  - Bubble Dynamics near Membrane*:

** A. Hajizadeh Aghdam, S.W. Ohl, B.C. Khoo, M.T. Shervani-Tabar, M.R.H. Nobari, Effect of the viscosity on the behavior of a single bubble near a membrane, Int. J. Multiphas. Flow 47 (2012) 17–24.*

Thank you for your attention.